



a. We can treat the fluid within sections A and B as the control volume. All equations will be carried out with respect to this boundary.

b. i) We can get the velocity of the oil at section B from the continuity equation.

$$m = \rho_A v_A A_A = \rho_B v_B A_B \text{ where } m = \text{mass, } \rho = \text{density, } v = \text{velocity, } A = \text{area}$$

Assuming density is constant:

$$v_B = \frac{v_A A_A}{A_B} = \frac{(3.25 \text{ m/s}) \left[ \frac{\pi}{4} (0.15 \text{ m})^2 \right]}{\left[ \frac{\pi}{4} (0.10 \text{ m})^2 \right]} = 7.3125 \text{ m/s}$$

Hence, the velocity of the oil at section B is **7.3125 m/s**

ii) To get the pressure at section B, we can use the Bernoulli equation.

$$g\Delta z + \Delta \frac{v^2}{2} + \Delta \frac{P}{\rho} = 0 \text{ where } z = \text{elevation, } v = \text{velocity, } \rho = \text{density, } P = \text{pressure}$$

Since the pipe is in a horizontal plane, the change in elevation is zero. The equation

then can be reduced to:

$$\Delta \frac{P}{\rho} = - \Delta \frac{v^2}{2}$$

$$\frac{P_B - P_A}{\rho} = - \left[ \frac{v_B^2 - v_A^2}{2} \right]$$

$$\frac{P_B - 147000 \text{ Pa}}{830 \text{ kg/m}^3} = - \frac{(7.3125 \text{ m/s})^2 - (3.25 \text{ m/s})^2}{2}$$

Solving for  $P_B$ , we get 129,192.2852 Pa or **129 kPa**

iii) To determine the magnitude and direction of the force required to hold the bend in place, we will apply momentum balance to the control volume (CV):

$$\begin{aligned} (\text{Sum of forces acting on CV}) &= (\text{Rate of momentum out of CV}) - \\ &(\text{Rate of momentum into CV}) + \\ &(\text{Rate of accumulation of momentum in CV}) \end{aligned}$$

Assuming steady state flow, the rate of accumulation of momentum can be set to zero.

$$\begin{aligned} \text{Thus, } (\text{Sum of forces acting on CV}) &= (\text{Rate of momentum out of CV}) - \\ &(\text{Rate of momentum into CV}) \end{aligned}$$

For this system, the force caused by gravity and friction will be neglected. The only forces acting are solid surface force (the force exerted by the pipe on the fluid) and pressure force (the force caused by the pressures acting on the surface of the control volume).

$$\Sigma F \text{ (Sum of forces acting on CV)} = R \text{ (solid surface force)} + F_p \text{ (pressure force)}$$

Thus, the force exerted by the pipe on the fluid can be expressed as:

$$R = \Sigma F - F_p = m(v_B - v_A) - (P_A A_A - P_B A_B)$$

Resolving into component forces:

$$R_x = m(v_B \cos 30 - v_A) - (P_A A_A - P_B A_B \cos 30)$$

$$R_y = m(v_B \sin 30) - (-P_B A_B \sin 30)$$

The mass of the oil  $m$  can be calculated from the continuity equation:

$$m = (830 \text{ kg/m}^3)(3.25 \text{ m/s}) \left[ \frac{\pi}{4} (0.15 \text{ m})^2 \right] = 47.66875978 \text{ kg/s}$$

Plugging values into  $R_x$  and  $R_y$  :

$$R_x = (47.6688 \text{ kg/s}) [(7.3125 \text{ m/s})\cos 30 - (3.25 \text{ m/s})] -$$

$$[(147000 \text{ Pa})(\frac{\pi}{4}(0.15 \text{ m})^2) - (129192 \text{ Pa})(\frac{\pi}{4}(0.10 \text{ m})^2)\cos 30]$$

$$= -1572.017342 \text{ N}$$

$$R_y = (47.6688 \text{ kg/s}) [(7.3125 \text{ m/s})\sin 30] - [-(129192 \text{ Pa})(\frac{\pi}{4}(0.10 \text{ m})^2)\sin 30]$$

$$= 681.6258204 \text{ N}$$

These are the components of the force exerted by the pipe on the fluid.

The components of the force exerted by the fluid on the pipe (reaction force) is the negative of  $R_x$  and  $R_y$  .

Hence, the magnitude and direction of the force required to hold the bend in place (reaction force) is:

**1572 N to the right and 682 N downward**